UNIT-3 Theory of logics

Proposition :- A Proposition or Statement is a declarative sentence that is either true or false, but not both.

Examples → 1. Delhi is the capital of India. (True)

2. Paris is in France. (True)
3. The sun rises in the west. (False)
4. $2+6=8$ (True)
5. $5+6>7$ (True)
6. $x+y=1$ (depend on x, y)
7. open the door (command)
8. what is the colour of Blackboard? (question)

The sentences (1), (2), (3) and (4) and (5) are Propositions whereas (6), (7) and (8) are not.

Note:- ① Questions, exclamations and commands are not Propositions.

② Propositions are Represented by $p, q, r, \ldots$ and called Propositional variables.

compound Proposition :→ A Proposition obtained from the combinations of two or more Propositions by means of logical operators or connectives of two or more Propositions or by negating a single Proposition is called composite or compound Proposition.

Connectives :→

The words and Phrases (or symbols) used to form compound Propositions are called connectives.

There are five basic connectives called Negation, Conjunction, disjunction, Conditional and Biconditional.

① **Negation:-** If p is any proposition, the negation of p, denoted by $\sim p$, $\neg p$ or p' is a proposition which is false when p is true and true when p is false. It is uniary operation.

e.g → if p: Paris is in france
then $\sim p$: Paris is not in france
or $\sim p$: It is not the case that Paris is in france.

Truth Table for $\sim p$:

p	$\sim p$
T	F
F	T

② **Conjunction:** → The conjuction of two statements p and q is the compound statement denoted by $p \wedge q$ and read as "p and q". $p \wedge q$ is true when both p and q are true, otherwise it is false.

e.g → ① p: It is cold q: It is raining
Then $p \wedge q$: It is cold and raining
② p: $5x+6=26$, q: $x>3$
Then $p \wedge q$: $5x+6=26$ and $x>3$

Truth Table for $p \wedge q$:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(3.) **Disjunction :—** The disjunction of two statements p and q is the compound statement denoted by $p \vee q$ and read as "p or q".
The statement $p \vee q$ is true if atleast one of p or q is true and false when both p and q are false.

e.g → if p: It is cold . q: It is raining
Then $p \vee q$: It is cold or raining

Truth Table for $p \vee q$:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(4.) **Conditional Proposition :→**
The conditional proposition of p and q is if p then q. read as 'p implies q' denoted by $p \to q$.
The proposition p is called antecedent or Hypothesis and proposition q is called Consequent or Conclusion.

e.g → (1.) if Today is monday then yesterday was Sunday.
(2.) if it rains then I will carry an umbrella.

The Truth table for $p \to q$:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

5. Biconditional Proposition: →

The Biconditional Proposition of p and q is p if and only if q (p iff q), denoted by $p \leftrightarrow q$

$p \leftrightarrow q$: if p then q and if q then p.

$p \leftrightarrow q \equiv$ p is necessary and sufficient condition for q.

e.g → (1) Ram eats if and only if the food is tasty

(2) He swims if and only if the water is warm.

Truth Table for $p \leftrightarrow q$:

$p \leftrightarrow q$ is true when both of p and q are either true or false.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Table :-

A Truth table is a table that shows the truth value (True or false) of a compound Proposition for all Possible cases.

Ex:- construct the truth Table for each compound Proposition

i.) $p \wedge (\sim q \vee q)$ ii.) $\sim (p \vee q) \vee (\sim p \wedge \sim q)$.

Solⁿ:- (i.) Truth Table for $p \wedge (\sim q \vee q)$ is given by

p	q	$\sim q$	$\sim q \vee q$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

(ii) The Truth table for $\sim(p \vee q) \vee (\sim p \wedge \sim q)$ is given by

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$	$\sim(p \vee q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	F	T	F	F	F
F	F	T	T	F	T	T	T

Converse, Contrapositive and Inverse of a Conditional Statement: →

Let p and q are two statements, Then the converse, contrapositive and Inverse of $p \to q$ is defined as

(1.) Converse:- The converse of $p \to q$ is $q \to p$

(2.) contrapositive:- The contrapositive of $p \to q$ is $\sim q \to \sim p$

(3.) Inverse:- The Inverse of $p \to q$ is $\sim p \to \sim q$.

Truth Table for $q \to p$, $\sim q \to \sim p$ & $\sim p \to \sim q$

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim q \to \sim p$	$\sim p \to \sim q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

Ex:-1 Find the converse, contrapositive and Inverse of "If It rains, then I will not go to market."

Sol^n:- Let p: It rains, q: I will go to market

Then $p \to \sim q$

converse: $\sim q \to p$: If I will not go to market, then It rains.

contrapositive: $q \to \sim p$: If I will go to market, then It does not rain.

Inverse: $\sim p \to q$: If It does not rain, then I will go to market.

Ex:-2 write the converse, contrapositive and Inverse of the following statements:

(i.) I am in trouble if the work is not finished on time.

2023 (ii.) if $x + 5 = 8$ Then $x = 3$.

Soln:-

i.) Let p: The work is finished on time.
and q: I am in trouble

Then the conditional statement is $\sim p \rightarrow q$

converse: $q \rightarrow \sim p$
if I am in trouble, then the work is not finished on time.

contrapositive: $\sim q \rightarrow p$
if I am not in trouble, Then the work is finished on time.

Inverse: $p \rightarrow \sim q$
if the work is finished on time. Then I am not in trouble.

ii.) let $p: x+5=8$ and $q: x=3$
Then conditional statement is $p \rightarrow q$

converse: $q \rightarrow p$
if $x = 3$ Then $x+5=8$

contrapositive: $\sim q \rightarrow \sim p$
if $x \neq 3$. Then $x+5 \neq 8$

Inverse: $\sim p \rightarrow \sim q$
if $x+5 \neq 8$, Then $x \neq 3$

Ques.1 Construct the truth table for the following statements

(2022) (i.) $(p \to \sim q) \to \sim p$ (ii.) $p \leftrightarrow (\sim p \vee \sim q)$

Solⁿ:-

i.) The truth Table for $(p \to \sim q) \to \sim p$ is given by

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$(p \to \sim q) \to \sim p$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	T	T

ii.) The truth table for $p \leftrightarrow (\sim p \vee \sim q)$ is given by

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \leftrightarrow (\sim p \vee \sim q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	T	F

Ques.2. Construct the truth table for the following Proposition: $((p \to q) \vee r) \vee (p \to q \to r)$

Solⁿ:- Truth table for the given Proposition is

p	q	r	$p \to q$	$(p \to q) \vee r$	$(p \to q \to r)$	$((p \to q) \vee r) \vee (p \to q \to r)$
T	T	T	T	T	T	T
T	T	F	T	T	F	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Tautology, Contradiction and Satisfiability :-

Tautology:- A compound Proposition that is always true for all Posible Truth Values of its variables or that contain only T in the last coloum of its truth table is called a Tautology.
It is denoted by T.

Contradiction:- A compound Proposition that is always false for all Posible Truth Values of its variables. or that contains only F in the last coloum of its truth table is called a contradiction.
F

Contingency:- A Proposition that is neither a Tautology nor a contradiction is called a contingency
i.e a compound Proposition that contain T as well as F in the last coloum of its truth table is called a contingency. or Satisfiability.

OR **Satisfiability:-** A compound Proposition is Satisfiable if it is true for some assignment of truth values to its Variables

Hence A Tautology is always Satisfiable.

Ex:- Use truth table to determine which of the following is a Tautology or a Contradiction:

i.) $\sim(q \to r) \wedge r \wedge (p \to q)$

ii.) $((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r$

Solⁿ:- i.) Let $P(p,q,r) := \sim(q \to r) \wedge r \wedge (p \to q)$

Then the truth table for $P(p,q,r)$:

p	q	r	$q \to r$	$\sim(q \to r)$	$p \to q$	$P(p,q,r)$
T	T	T	T	F	T	F
T	T	F	F	T	T	F
T	F	T	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	F
F	T	F	F	T	T	F
F	F	T	T	F	T	F
F	F	F	T	F	T	F

Since the truth values of $P(p,q,r)$ for all possible cases of its variables are false which can be seen in the last column of truth table

So $P(p,q,r)$ is a contradiction.

ii.) Let $Q(p,q,r) := ((p \vee q) \wedge (p \to r) \wedge (q \to r))$

and $R(p,q,r) := ((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r$

Then the truth table for $R(p,q,r)$:

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$Q(p,q,r)$	$R(p,q,r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

So $R(p,q,r)$ is a tautology.

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Ques. 1 check whether $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$ is a Tautology, contradiction or a contingency.

Sol^n:- Let $P(p, q, r) := (p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$

p	q	r	$\sim p$	$p \vee q$	$\sim p \vee r$	$q \vee r$	$(p \vee q) \wedge (\sim p \vee r)$	$P(p, q, r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	T	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	T	T	F	T
F	F	F	T	F	T	F	F	T

Since all the entries in the last coloum of truth table for Proposition $P(p, q, r)$ are T (truth value T) So the given Proposition $P(p, q, r)$ is a Tautalogy.

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Ques. 2 Use truth table to identify whether $(p \wedge q) \rightarrow (p \vee q)$ is a taublogy or contradiction.

Sol^n:- The Truth table for $(p \wedge q) \rightarrow (p \vee q)$:

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Since all the enkies in the last coloum of Truth table for Proposition $(p \wedge q) \rightarrow (p \vee q)$ are T So the given Proposition is a Tautalogy.

Logical Equivalence :→

If two compound Propositions $P(p,q,r)$ and $Q(p,q,r)$, where $p, q, r, \ldots$ are Propositional variables, have the same truth values in every possible case or if $P \leftrightarrow Q$ is a tautology. Then the propositions are logically equivalent or simply equivalent and denoted by $P(p,q,r,\ldots) \equiv Q(p,q,r,\ldots)$

or $P \Leftrightarrow Q$

Algebra of Propositions :→

(1.) Idempotent laws :→ (a) $p \vee p \equiv p$
(b) $p \wedge p \equiv p$

(2.) Associative laws :→ (a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$
(b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(3.) commutative laws :→ (a) $p \vee q \equiv q \vee p$
(b) $p \wedge q \equiv q \wedge p$

(4.) Distributive laws :→ (a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
(b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(5.) Identity laws :→ (a) $p \vee F \equiv p$, $p \wedge T \equiv p$
(b) $p \vee T \equiv T$, $p \wedge F \equiv F$

(6.) complement laws :– (a) $p \vee \sim p \equiv T$, $p \wedge \sim p \equiv F$
(b) $\sim T \equiv F$, $\sim F \equiv T$

(7.) Involution law :– $\sim(\sim p) \equiv p$

(8.) De Morgan's laws :– (a) $\sim(p \vee q) \equiv \sim p \wedge \sim q$
(b) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

(9.) Absorption laws :– (a) $p \vee (p \wedge q) \equiv p$
(b) $p \wedge (p \vee q) \equiv p$

Def:- A Proposition p logically implies q iff $p \Rightarrow q$ is a tautology. In this case, we write $p \rightarrow q$. Further, Two Propositions p and q are logically equivalent iff $p \Leftrightarrow q$ is a tautology. In this case we write $p \leftrightarrow q$.

Theorem:- if $(p_1 \wedge p_2 \wedge \cdots \wedge p_n \wedge p) \Rightarrow q$ Then $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \Rightarrow (p \rightarrow q)$

Ex:- Use truth table to show that $p \rightarrow q \equiv \sim p \vee q \equiv \sim q \rightarrow \sim p$.

Solⁿ:-1 The truth table for $p \rightarrow q$, $\sim p \vee q$, and $\sim q \rightarrow \sim p$:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim p \vee q$	$\sim q \rightarrow \sim p$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

The fifth and sixth coloums of the table Proves the equivalence $p \rightarrow q \equiv \sim p \vee q$. (Implication as Disjunction) and the sixth and seventh coloums of the table Proves the equivalence $\sim p \vee q \equiv \sim q \rightarrow \sim p$

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Ex:-2 Show that the statements $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are equivalent.

Solⁿ:- The truth table for $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$:

p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

The fifth and last coloum shows that
$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q) \equiv \sim(p \vee q) \vee (p \wedge q)$

By De morgan's law

Ques. without using truth table, Prove the following

i.) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

ii.) $((p \wedge q \wedge a) \to r) \wedge (a \to (p \vee q \vee r)) \equiv ((p \leftrightarrow q) \wedge a) \to r$

Solⁿ:- i.) we have,

$(p \to r) \wedge (q \to r) \equiv (\sim p \vee r) \wedge (\sim q \vee r)$ (Implicatⁿ as disjunction)

$\equiv (\sim p \wedge \sim q) \vee r$ (Distributivity)

$\equiv \sim(p \vee q) \vee r$ (De morgan's)

$\equiv (p \vee q) \to r$ (Implication as disjunctⁿ)

ii.) $((p \wedge q \wedge a) \to r) \wedge (a \to (p \vee q \vee r))$

$\equiv (\sim(p \wedge q \wedge a) \vee r) \wedge (\sim a \vee (p \vee q \vee r))$

$\equiv ((\sim p \vee \sim q \vee \sim a) \vee r) \wedge ((\sim a \vee p \vee q) \vee r)$ De morgan

$\equiv ((\sim p \vee \sim q \vee \sim a) \vee r) \wedge ((p \vee q \vee \sim a) \vee r)$ commutative

$\equiv ((\sim p \vee \sim q \vee \sim a) \wedge (p \vee q \vee \sim a)) \vee r$ distributivity

$\equiv (((\sim p \vee \sim q) \wedge (p \vee q)) \vee \sim a) \vee r$ distributivity

$\equiv (\sim [(p \wedge q) \vee (\sim p \wedge \sim q)] \vee \sim a) \vee r$ De Morgan

$\equiv \sim([(p \wedge q) \vee (\sim p \wedge \sim q)] \wedge a) \vee r$ De Morgan

$\equiv \sim((p \leftrightarrow q) \wedge a) \vee r$ $\because p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

$\equiv ((p \leftrightarrow q) \wedge a) \to r$

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Tautologically Implication : →

A compound Proposition $A(p, q, r, \dots)$ is said to be tautologically imply or simply imply the compound Proposition $B(p, q, r, \dots)$ if and only if $A \rightarrow B$ is a tautology. It is denoted by $A \Rightarrow B$

Ques. Prove the following fundamental Implications

i.) Modus Ponens : $(p \rightarrow q) \wedge p \Rightarrow q$.

ii.) Modus Tollens : $(p \rightarrow q) \wedge \sim q \Rightarrow \sim p$

iii.) Hypothetical Syllogism : $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$

iv.) Disjunctive Syllogism : $(p \vee q) \wedge \sim p \Rightarrow q$.

Sol^n :- (i.) We have

$$
\begin{aligned}
(p \rightarrow q) \wedge p \rightarrow q &\equiv (\sim p \vee q) \wedge p \rightarrow q && \text{Implication as Disjunction} \\
&\equiv ((\sim p \wedge p) \vee (q \wedge p)) \rightarrow q && \text{Distributivity} \\
&\equiv (F \vee (q \wedge p)) \rightarrow q && \text{complementation} \\
&\equiv (q \wedge p) \rightarrow q && \text{Identity law} \\
&\equiv \sim (q \wedge p) \vee q && \text{Implication as Disjunction} \\
&\equiv (\sim q \vee \sim p) \vee q && \text{De Morgan's law} \\
&\equiv (\sim q \vee q) \vee \sim p && \text{Commutativity} \\
&\equiv T \vee \sim p && \text{complementation} \\
&\equiv T && \text{Identity law}
\end{aligned}
$$

Therefore, $(p \rightarrow q) \wedge p \rightarrow q$ is a Tautology and $(p \rightarrow q) \wedge p \Rightarrow q$ is a tautologically Implication.

ii.) Modus Tollens

$(p \to q) \land \sim q \to \sim p \equiv (\sim p \lor q) \land \sim q \to \sim p$ Implicat^n as Disjunct^n

$\equiv ((\sim p \land \sim q) \lor (q \land \sim q)) \to \sim p$ Distributivity

$\equiv (\sim (p \lor q) \lor F) \to \sim p$ De Morgan's & complement law

$\equiv \sim (p \lor q) \to \sim p$ Identity law

$\equiv (p \lor q) \lor \sim p$ Implication as Disjunction

$\equiv (p \lor \sim p) \lor q$ commutativity

$\equiv T \lor q$ complement law

$\equiv T$ dominance

Therefore, $(p \to q) \land \sim q \to \sim p$ is a Tautology.
and Hence $(p \to q) \land \sim q \Rightarrow \sim p$ is a Tautologically Implication.

iii.) Hypothetical Syllogism

$(p \to q) \land (q \to r) \equiv (\sim p \lor q) \land (\sim q \lor r)$ Implicat^n as Disjunction

$\equiv (\sim p \land (\sim q \lor r)) \lor (q \land (\sim q \lor r))$ Distributivity

$\equiv ((\sim p \land \sim q) \lor (\sim p \land r)) \lor ((q \land \sim q) \lor (q \land r))$ "

$\equiv ((\sim p \land \sim q) \lor (\sim p \land r)) \lor (F \lor (q \land r))$ complement law

$\equiv ((\sim p \land \sim q) \lor (\sim p \land r)) \lor (q \land r)$ Identity law

$\equiv ((\sim p \land \sim q) \lor (q \land r)) \lor (\sim p \land r)$ Associative & commutative

$\equiv ((\sim p \land \sim q) \lor q) \land ((\sim p \land \sim q) \lor r)) \lor (\sim p \land r)$

$\equiv (((\sim p \lor q) \land (\sim q \lor q)) \land ((\sim p \lor r) \land (\sim q \lor r))) \lor (\sim p \land r)$

$\equiv (((\sim p \lor q) \land T) \land (\sim p \lor r) \land (\sim q \lor r)) \lor (\sim p \land r)$

$\equiv ((\sim p \lor q) \land (\sim p \lor r) \land (\sim q \lor r)) \lor (\sim p \land r)$

$\equiv ((\sim p \lor q) \lor (\sim p \land r)) \land ((\sim p \lor r) \lor (\sim p \land r)) \land ((\sim q \lor r) \lor (\sim p \land r))$

$\equiv (\sim p \vee q) \wedge (\sim p \vee r) \wedge (\sim q \vee r)$

$\equiv (\sim p \vee q) \wedge ((\sim p \wedge \sim q) \vee r))$ distributivity

$\equiv (\sim p \vee q) \wedge (\sim (p \vee q) \vee r)$ De morgan

$\equiv (p \to q) \wedge ((p \vee q) \to r)$ Implicatn as Disjunctn

$\equiv p \to q$

where

$(\sim p \vee q) \vee (\sim p \wedge r) \equiv ((\sim p \vee q) \vee \sim p) \wedge ((\sim p \vee q) \vee r)$

$\equiv (\sim p \vee q) \wedge (\sim p \vee q \vee r)$

$\equiv \sim p \vee q$

$(\sim q \vee r) \vee (\sim p \wedge r) \equiv ((\sim q \vee r) \vee \sim p) \wedge ((\sim q \vee r) \vee r)$

$\equiv (\sim q \vee r \vee \sim p) \wedge (\sim q \vee r)$

$\equiv \sim q \vee r$

Therefore, $(p \to q) \wedge (q \to r) \to (p \to r)$ is a Tautology.

3.) Disjunctive syllogism

we have

$((p \vee q) \wedge \sim q) \to p \equiv ((p \wedge \sim q) \vee (q \wedge \sim q)) \to p$ Distributive law

$\equiv ((p \wedge \sim q) \vee F) \to p$ complement law

$\equiv (p \wedge \sim q) \to p$ Identity law

$\equiv \sim (p \wedge \sim q) \vee p$ Implicatn as Disj.

$\equiv (\sim p \vee q) \vee p$ De morgan's law

$\equiv (\sim p \vee p) \vee q$ commutativity

Identity law

$\equiv T \vee q$ dominance.

$\equiv T$

Therefore $(p \vee q) \wedge \sim q \to p$ is a Tautologically Implication.

Argument :→

An argument is a process by which a conclusion is drawn from a set of Propositions.

Premises or Hypothesis : The given set of Propositions

Conclusion : The final Proposition derived from the given Propositions

Mathematically, An argument can be written as

$$\left.\begin{array}{l} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{array}\right\} \text{Premises}$$

$$\therefore q \longrightarrow \text{conclusion.}$$

Valid Argument: An argument is said to be logically valid iff $(p_1 \wedge p_2 \wedge p_3 \cdots \wedge p_n) \rightarrow q$ is a tautology. i.e. if the Premises are all true, the conclusion must be be true.

Ex:- An Argument is given as :

If you study hard, Then you get A's grade

If you study hard

∴ you get A's grade.

Represent the argument Symbolically and determine whether the argument is valid.

Soln:- Let p: you study hard

q: you get A's grade

Then Symbolically, the Argument can be written as

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

For Validity of argument, we have to show that $(p \to q) \wedge p \to q$ is a Tautology

p	q	$p \to q$	$(p \to q) \wedge p$	$(p \to q) \wedge p \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since $(p \to q) \wedge p \to q$ is a tautology
Hence the given argument is Valid.

Rules of Inference :→

Rules of Inference are criteria for determing the Validity of an argument. Any conclusion which is arrived by following rules is called a Valid conclusion, and the argument is called Valid Argument.

(1) **Modus Ponens** :– If the statements p and $p \to q$ are accepted as true, Then q must be true.
symbolically, An argument

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

This is the first Fundamental Rule

is Valid.
or $(p \to q) \wedge p \to q$ is a tautology.

(2) **Modus Tollens** :– if statements $p \to q$ and $\sim q$ are true, then $\sim p$ must be true
symbolically, An argument of the form

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

is valid

or $(p \to q) \wedge \sim q \to \sim p$ is a tautology.

3.) Hypothetical Syllogism :—

Whenever the two implications $p \to q$ and $q \to r$ are accepted as true, then implication $p \to r$ is also accepted as true.

Symbolically, It can be represented as

$$\begin{array}{c} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

This argument is known as Hypothetical syllogism or Second fundamental Rule.

Its tautological form $[(p \to q) \wedge (q \to r)] \to (p \to r)$

4.) Disjunctive Syllogism :—

An argument of the form

$$\begin{array}{c} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$ is valid

This Argument states that when there are two Possibilities and one can rule one out, the other must be the Case.

Rule-P, Rule-T and Rule-CP of logically deductive Reasoning :—

Rule-P states that a Premise may be introduced as an argument at any step.

Rule-T states that a statement formula may be introduced as an argument at any step, subject to it is tautologically implied by one or more Preceding formulas.

Rule-CP states that if a formula s can be derived from another formula r, together with a set of Premises P, then the statement $r \to s$ can be derived from the set of Premises alone.

i.e in Symbolically form, $p \to (r \to s) \equiv (p \wedge r) \to s$

Nine Inference Rules with its Tautolical form

Name	Rule of Inference	Tautological form
Addition	a.) $\frac{p}{\therefore p \vee q}$ b.) $\frac{q}{\therefore p \vee q}$	a.) $p \rightarrow (p \vee q)$ b.) $q \rightarrow (p \vee q)$
Simplification	a.) $\frac{p \wedge q}{\therefore p}$ b.) $\frac{p \wedge q}{\therefore q}$	a.) $(p \wedge q) \rightarrow p$ b.) $(p \wedge q) \rightarrow q$
Conjunction	p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$
Modus Ponens	$p \rightarrow q$ p $\therefore q$	$[(p \rightarrow q) \wedge p] \rightarrow q$
Modus Tollens	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
Hypothetical Syllogism	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
Disjunctive Syllogism	a.) $p \vee q$, $\sim p$, $\therefore q$ b.) $p \vee q$, $\sim q$, $\therefore p$	$[(p \vee q) \wedge \sim p] \rightarrow q$ $[(p \vee q) \wedge \sim q] \rightarrow p$
Constructive dilemma	$p \rightarrow q \wedge (r \rightarrow s)$ $p \vee r$ $\therefore q \vee s$	$(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r) \rightarrow (q \vee s)$
Destructive Dilemma	$p \rightarrow q \wedge (r \rightarrow s)$ $\sim q \vee \sim s$ $\therefore \sim p \vee \sim r$	$(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim q \vee \sim s) \rightarrow (\sim p \vee \sim r)$

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Ques. 1. Check the validity of the Argument:
If I try hard, and I have talent, Then I will become a Scientist. If I become a Scientist, Then I will be happy. Therefore, if I will not be happy, Then I did't try hard or I do not have talent.

Sol^n:- Let p: I try hard q: I have talent
r: I will become a Scientist
s: I will be happy.

Symbolically, The given argument can be written as

$$\begin{array}{c} p \wedge q \rightarrow r \\ r \rightarrow s \\ \hline \therefore \sim s \rightarrow \sim p \vee \sim q \end{array}$$

So we have

1.) $p \wedge q \rightarrow r$ Rule-P (given Premise)
2.) $r \rightarrow s$ Rule-P
3.) $p \wedge q \rightarrow s$ Hypothetical Syllogism on (1), (2)
4.) $\sim s \rightarrow \sim (p \wedge q)$ Rule-T (contrapositive) on (3)
5.) $\sim s \rightarrow \sim p \vee \sim q$ Rule-T (De Morgan's law) on (4)

Therefore, The argument is Valid.

Ques. 2 Check the validity of the argument:
If the traffic is heavy, Then travelling is difficult. If customers arrived on time, then travelling was not difficult. They arrived on time. Therefore, the traffic was not heavy.

Sol^n:- Let p: the traffic is heavy, q: travelling is difficult
r: customers arrived on time.

Symbolically, The given argument can be written as

$$\begin{array}{c} p \to q \\ r \to \sim q \\ r \\ \hline \therefore \sim p \end{array}$$

we have

1.) $p \to q$ Rule-P

2.) $\sim q \to \sim p$ Rule-T (contrapositive) on (1)

3.) $r \to \sim q$ Rule-P

4.) $r \to \sim p$ Hypothetical Syllogism (3), (2)

5.) r Rule-P

6.) $\sim p$ Modus Ponens (4), (5)

Therefore, The argument is valid.

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Ques.3. Justify the validity of the following argument:

If I get the job and work hard, then I will get Promoted. If I get Promoted, Then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard.

Solⁿ:- Let p: I get the job, q: I work hard

r: I will get Promoted

s: I will be happy.

Then, The given argument can be written in sybal form as

$$\begin{array}{c} (p \wedge q) \to r \\ r \to s \\ \sim s \\ \hline \therefore \sim p \vee \sim q \end{array}$$

we have

1.) $p \wedge q \to r$ given Premise

2.) $r \to s$ given Premise

3.) $p \wedge q \to s$ Hypothetical Syllogism on (1), (2)

4.) $\sim s$ given Premise

5.) $\sim(p \wedge q)$ Modus Tollens on (3), (4)

6.) $\sim p \vee \sim q$ De morgan's on (5)

Therefore the given argument is valid.

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Ques. 4. Justify the Validity of the following argument:

If Mary runs for office, she will be elected. If mary attends the meeting, she will run for office. Either mary will attend the meeting or She will go to India But mary can not go to India. Thus mary will be elected.

Soln:- Let p: mary runs for office

q: mary will be elected

r: mary attends the meeting

s: mary will go to India

Then the given argument can be written in Symbolic form as

$$\begin{array}{l} p \to q \\ r \to p \\ r \vee s \\ \sim s \\ \hline \therefore q \end{array}$$

we have

1.) $r \rightarrow p$ given Premise (Rule-P)

2.) $p \rightarrow q$ given Premise (Rule-P)

3.) $r \rightarrow q$ Hypothetical Syllogism on (1), (2)

4.) $r \vee s$ Rule-P

5.) $\sim s$ Rule-P

6.) r disjunctive Syllogism on (4), (5)

(7.) q Modus Ponens on (3), (6)

Therefore, The given argument is valid.

Consistent and Inconsistent Premises:—

Consistent Premises: A set of Premises $P_1, P_2, \ldots, P_n$ are said to be consistent if their conjunction $P_1 \wedge P_2 \wedge \ldots \wedge P_n$ has truth value T in atleast one Possible case of their variables.

Inconsistent Premises: A set of Premises $P_1, P_2, \ldots, P_n$ are said to be Inconsistent if their conjunction $P_1 \wedge P_2 \wedge \ldots \wedge P_n$ has the truth value F in every Possible case of their variables.

OR

A set of Premises $P_1, P_2, \ldots, P_n$ are said to be Inconsistent if $P_1 \wedge P_2 \ldots \wedge P_n \rightarrow F$ i.e they lead to contradiction. otherwise the Premises are consistent.

Ques.1 Justify the Premises $r \to \sim q$, $r \vee s$, $s \to \sim q$, $p \to q \Rightarrow \sim p$ are consistent.

Solⁿ:- we have to show that the Premises $r \to \sim q$, $r \vee s$, $s \to \sim q$ and $p \to q$ lead to $\sim p$

we have

1.) $p \to q$ Rule-P

2.) $r \to \sim q$ Rule-P

3.) $q \to \sim r$ Rule-T (contrapositive) on (2)

4.) $p \to \sim r$ Hypothetical Syllogism on (1), (3)

5.) $s \to \sim q$ Rule-P

6.) $q \to \sim s$ Rule-T (contrapositive) on (5)

7.) $p \to \sim s$ Hypothetical syllogism on (1), (6)

8.) $r \to \sim p$ Rule-T (contrapositive), (4)

9.) $s \to \sim p$ Rule-T (contrapositive), (7)

10.) $(r \to \sim p) \wedge (s \to \sim p)$ Conjunction (8), (9)

11.) $r \vee s$ Rule-P

12.) $\sim p \vee \sim p$ constructive dilemma (10), (11)

13.) $\sim p$ Idempotent, (12)

Hence the given set of Premises are Consistent.

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Ques. 2 Show that the Premises $p \to q$, $p \to r$, $q \to \sim r$, p are inconsistent.

Solⁿ:- To show $p \to q$, $p \to r$, $q \to \sim r$, p are Inconsistent, we will show that these Premises lead to a contradiction.

we have,

1.)	p	Rule - P
2.)	$p \to q$	Rule - P
3.)	q	Modus Ponens on (2), (1)
4.)	$q \to \sim r$	Rule - P
5.)	$\sim r$	Modus Ponens on (4), (3)
6.)	$p \to r$	Rule - P
7.)	$\sim r \to \sim p$	or Rule - T (contrapositive of (6))
8.)	$\sim p$	Modus Ponens on (7), (5)
9.)	$p \wedge \sim p \equiv F$	(1), (8)

Hence, the given Set of Premises are Inconsistent.

First order logic :→

First order logic (FOL) is used to express any types of mathematical relations or sentences written in a natural language.

Predicate Calculus :-

Predicate :- A Predicate P in certain finite number of variables $x_1, x_2, \cdots x_n$ is a property or a relation that these variables satisfy.

the n-variable predicate written as $P(x_1, x_2, \cdots x_n)$.

Ex :- $B(x)$: x is a bachelor.

where x is predicate variable

and $B(x)$ is prepositional function.

because each choice of x produces a proposition $P(x)$ that is either true or false.

Universe of discourse / Domain :-

Let $P(x_1, \cdots, x_n)$ be an n-variable predicate. The set of values $x_1, \cdots, x_n$ may take is called a universe of discourse or simply universe of the predicate P

e.g → The domain for $B(x)$: x is a Bachelor can be taken as the set of all human names

→ The Predicate $L(x, y)$: "x loves y" is a two variable Predicate.

Quantifiers :-

Quantifiers are words that refer to quantities such as some, few, many, all, none and indicate how frequently a certain statement is true.

Universal quantifier: — The Phrase "for all" (denoted by $\forall$) is called the universal quantifier.

e.g → ① "All human beings are mortal."

let $P(x)$: x is human being

$Q(x)$: x is mortal.

Then above Sentence Can be written as

$$\forall x\, [P(x) \rightarrow Q(x)]$$

② All birds can fly as

$$\forall x\, [Bird(x) \rightarrow fly(x)].$$

Existential quantifier: —

The Phrase "there exists" (denoted by $\exists$) is called the existential quantifier.

e.g → "there exists x Such that $x^2 = 5$"

Let $P(x)$: $x^2 = 5$

Then $\exists x\, P(x)$.

→ In terms of two variables,

The Sentence "Everbody loves Somebody"

let $L(x, y)$: x loves y

Then above Sentence Can be written as

$$\forall x\, \exists y\, L(x, y).$$

Nested Predicate: — A statement or Sentence is Called a nested Predicate if it involves Predicates, Standard logical Connectives, and the two Universal quantifiers.

Ques.1 Convert the following statements in quantified expressions of Prediate logic.

i.) Not all birds can fly

ii.) Everbody loves somebody, iii.) Some mens are genius.

iii.) There is Somebody whom no one loves

iv.) No student has done every Problem in the assignment.

vi) Every real no. is either negative or has a Square root.

vii.) There is a barbar who shaves all men in the town who do not shave themselves.

viii) All integers are either even or odd.

Solⁿ:- i.) Let $B(x)$: x is a Bird

$F(x)$: x can fly

Then, the statement "All birds can fly" may be expressed as

$$\forall x\ [B(x) \rightarrow F(x)]$$

∴ the negation of "All birds can fly" is "Not all birds can fly" may be written as

$$\sim\{\forall x\ [B(x) \rightarrow F(x)]\} \equiv \exists x [B(x) \wedge \sim F(x)]$$

ii.) Every body loves somebody

Let $L(x, y)$: x loves y

∴ The given sentence can be written as

$$\forall x\ \exists y\ L(x, y)$$

iii.) Let $M(x)$: x is a man

$G(x)$: x is Genius

Then the statement "some mens are genius" can be expressed as

$$\exists x [M(x) \longrightarrow G(x)]$$

It can also be expressed as $\exists x\ G(x)$,
Provided the universe is taken as the set of all men

iv.) There is somebody whom no one loves

Let $L(x, y)$: x loves y

Then the given statement can be expressed as

$$\sim \{[\forall x \exists y\ L(x, y)]\} \equiv \exists x\ \forall y \sim L(x, y)$$

v.) No student has done every problem in the assignment.

Let $D(x, y)$: x has done the problem y of the assignment.

Then the given statement can be expressed as

$$\sim [\exists x\ \forall y\ D(x, y))$$

vi.) Every real no. is either negative or has a square root.

Let $N(x)$: x is negative

$S(x)$: x has a square root

Then the given statement can be expressed as

$$\forall x\ [N(x) \vee S(x)]$$

where the universe for x are real numbers.

vii.) There is a barber who shaves all men in the town who do not shave themselves.

Let $B(y)$: y is a Barber

$S(x, y)$: x shaves y

So the statement can be expressed as

$$\exists y\ (B(y) \wedge \forall [\sim S(x, x) \rightarrow S(y, x)])$$

where the universe for both x & y are all men.

if $y = b$ (barber) then

$$\exists b\ \forall x\ [\sim (S, x) \rightarrow S(b, x)]$$

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viii.) All Integers are either even or odd.

Let $I(x)$: x is even

$O(x)$: x is odd

Then the given statement can be expressed as

$$\forall x\ [I(x) \vee O(x)]$$

where the universe for x are real numbers.

Theory of Inference :—

Rules of specification : Rule - US

$\forall x\, P(x)$ is true iff $P(x)$ is true for every x in the universe of discourse, the Implication $x P(x) \Rightarrow P(y)$ holds. This is known as the rule of universal specification. where $P(y)$ is a proposition and y is the universe.

Rule of Universal Generalization : Rule - UG

when we have $P(x)$ is true for all x, it follows that $\forall x\, P(x)$ is true. That is $P(x) \Rightarrow \forall y\, P(y)$ holds. This is known as rule of universal generalization (or Rule - UG)

Rule of Existential specification : Rule - ES

if $(\exists x) P(x)$ is true, then $P(x)$ is true for some in the universe, and we have the Implication $(\exists x) P(x) \Rightarrow P(a)$. This is known as Rule - ES

Rule of Existential Generalization ; Rule - EG

if $P(x)$ is true for atleast one subject $x = a$ in the universe of discourse, Then we have the conclusion $P(a) \Rightarrow (\exists x) P(x)$. This is known as Rule - EG

First order Logic Equivalent Propositional functions:-

Let S and T be two Propositional functions involving Predicates and quantifiers. Then S and T are FOL Equivalent, denoted by $S \equiv T$ if they have the same truth value no matter which Predicates are substituted for these statements, and which domain of discourse is used for the involved Predicate variables.

E.g → $\forall x [P(x) \wedge Q(x)] \equiv \forall x P(x) \wedge \forall x Q(x)$.

Ex:-1 Show that

i.) $(\forall x P(x)) \vee (\forall x Q(x)) \Rightarrow \forall x [P(x) \vee Q(x)]$

ii.) $\exists x [P(x) \wedge Q(x)] \Rightarrow (\exists x P(x)) \wedge (\exists y Q(y))$

Solⁿ:- i.) Suppose P and Q are two Predicates with common domain of discourse s.t $(\forall x P(x)) \vee (\forall x Q(x))$ is true.

Then $P(x)$ is true for every x or $Q(x)$ is true for every x.

It is Possible to have some x in the domain for which both $P(x)$ and $Q(x)$ are true.

In any case all must satisfy $P(x)$ or $Q(x)$ or both are true. Hence, $\forall x [P(x) \vee Q(x)]$ is true.

ii.) Suppose P and Q are two Predicates with common domain such that $\exists x [P(x) \wedge Q(x)]$ is true. That is there is some x for which $P(x)$ is true and $Q(x)$ is true.

Therefore, for some a in the domain, $P(a) \wedge Q(a)$ is true.

Hence, by Rule-EG, $(\exists x P(x)) \wedge (\exists y Q(y))$ is true.

Ex:- Show that $\forall x\,(P(x) \vee Q(x)) \rightarrow \forall x\, P(x) \vee \exists\, Q(x)$ using Indirect method.

Sol^n:- In Indirect method, we assume $\sim(\forall x\, P(x) \vee \exists x\, Q(x))$ as the additional premise and prove contradiction F

1. $\sim(\forall x\, P(x) \vee \exists x\, Q(x))$ Additional Premise
2. $\sim(\forall x\, P(x)) \wedge \sim(\exists x\, Q(x))$ (1), De Morgan's Rule
3. $\exists x\,(\sim P(x)) \wedge \forall x\,(\sim Q(x))$ (2), Negation Equivalence
4. $\exists x\,(\sim P(x))$ (3), Simplification
5. $\forall x\,(\sim Q(x))$ (3), Simplification
6. $\sim P(a)$ (4), EI
7. $\sim Q(a)$ (5), UI
8. $\sim P(a) \wedge \sim Q(a)$ (6), (7) and Conjunction
9. $\sim (P(a) \vee Q(a))$ (8), De Morgan's law
10. $\forall x\,(P(x) \vee Q(x))$ Premise (given)
11. $P(a) \vee Q(a)$ (10) and UI
12. $(P(a) \vee Q(a)) \wedge \sim(P(a) \vee Q(a))$ (9), (11) and Conjunct^n
13. F (12), Negation law

Hence Proved.

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Ques. Translate the following statement in Symbolic form: All flowers are beautiful.

Soln:- let $F(x)$: x is a flower

$B(x)$: x is beautiful

Then the given statement may be expressed as

$$\forall x\,[F(x) \rightarrow B(x)]$$

where the universe of x is the set of all flowers